Statistics

Lecture 9

Feb 19-8:47 AM

Suppose prob. that any student is in favor of online classes is .8. (SG 16)

I randomly selected 400 students. Assume success is to be in favor of online classes.

1) $n = 400$

2) $P = .8$

3) $q = 1 - P = .2$

4) $\mu = np = 400(.8) = 320$

5) $\sigma^2 = npq$
$= 400(.8)(.2)$
$= 64$

6) $\sigma = \sqrt{\sigma^2}$
$= \sqrt{64} = 8$

7) Usual Range $\Rightarrow$ 95% Range $\Rightarrow \mu \pm 2\sigma$
$= 320 \pm 2(8)$
$= 320 \pm 16$
$\Rightarrow \boxed{304 \text{ To } 336}$

Apr 9-6:50 PM

8) Find the prob. that $\boxed{\text{exactly } 325}$ of them are in favor of online classes

$P(x=325) = \text{binompdf}(400, .8, 325) = \boxed{.042}$

9) Find the prob. that $\boxed{\text{at most } 330}$ of them are in favor of online classes.

$P(x \le 330) = \text{binomcdf}(400, .8, 330) = \boxed{.907}$

10) Find the Prob. that $\boxed{\text{at least } 310}$ of them are in favor of online classes.

$P(x \ge 310) = 1 - P(x \le 309) = 1 - \text{binomcdf}(400, .8, 309)$

we don't want | we want

309 310

$= \boxed{.904}$

11) Find the Prob. that between 304 and 336 of them, inclusive, are in favor of online classes.

$P(304 \le x \le 336) = P(x \le 336) - P(x \le 303)$

$= \text{binomcdf}(400, .8, 336) - \text{binomcdf}(400, .8, 303) = \boxed{.961}$

Usual Range

304 – 336

95% Range

SG 16 ✓

$\approx 96\%$

Apr 9-6:56 PM

1) Geometric Prob. dist.

SG 17

2) Poisson Prob. dist.

Geometric Prob. dist

It is Similar To binomial Prob. dist with

$p \to$ Prob. of Success , $q \to$ Prob. of failure

$p + q = 1$

Trials or events are independent

p & q remain unchanged for any trial.

There is no fixed number of trials

No n

$x \to$ # of trial when first success happens.

$P(x) = p \cdot q^{x-1} \quad , x \ge 1$

$\mu = \frac{1}{p} \quad , \quad \sigma^2 = \frac{q}{p^2} \quad , \quad \sigma = \sqrt{\sigma^2}$

Apr 9-7:09 PM

Consider a geometric Prob. dist with $p = .5$

$q = 1 - p = 1 - .5 = .5$

$\mu = \frac{1}{p} = \frac{1}{.5} = 2$

$\sigma = \sqrt{\sigma^2} = \sqrt{2} = 1.414 \approx 1$

$\sigma^2 = \frac{q}{p^2} = \frac{.5}{.5^2} = 2$

68% Range

$\mu \pm \sigma = 2 \pm 1 \Rightarrow \boxed{1 \text{ to } 3}$

P(First success happens on 3rd attempt)

$= P(x=3) = (.5)(.5)^{3-1} = \boxed{.125}$

$P(x) = p \cdot q^{x-1}$

using TI : $\boxed{\text{2nd}}$ $\boxed{\text{VARS}}$ $\boxed{\text{geometpdf}}$

$p = .5,\ x = 3$

$P(x=3) = \text{geometpdf}(.5, 3) = \boxed{.125}$

Find the Prob. that first success happens before the 4th attempt.

$P(x < 4) = P(x \le 3) = \text{geometcdf}(.5, 3) = \boxed{.875}$

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Prob. of hit for baseball players at bat is .25.

$p = .25$

$q = 1 - .25 = .75$

$\mu = \frac{1}{p} = \frac{1}{.25} = 4$

$\sigma = \sqrt{\sigma^2} = \sqrt{12} \approx 3.5$

Usual Range

$\sigma^2 = \frac{q}{p^2} = \frac{.75}{.25^2} = 12$

$\mu \pm 2\sigma$

$= 4 \pm 2(3.5) = 4 \pm 7$

$\Rightarrow \boxed{-3 \text{ to } 11}$

Prob. that first hit happens after the 4th at bat.

$P(x > 4) = P(x \ge 5) = 1 - P(x \le 4)$

we don't want | we want

want 4 5

$= 1 - \text{geometcdf}(.25, 4)$

$= \boxed{.316}$

Apr 9-7:25 PM

Poisson Prob. dist.:

μ is the average # of Successes in a given interval

$x \rightarrow$ # of Successes in that interval

$\rightarrow x = 0, 1, 2, 3, \ldots$

$$P(x) = \frac{\mu^x}{x!} \cdot e^{-\mu}$$

lambda

$e \approx 2.718$

$\sigma^2 = \mu$

$\sigma = \sqrt{\sigma^2}$

Poisson pdf (λ, x)

Poissoncdf (λ, x)

Apr 9-7:32 PM

Consider a Poisson Prob. dist with the average of 16 Successes in a fixed interval.

$\mu = 16$ $\quad \sigma^2 = \mu = 16$ $\quad \sigma = \sqrt{\sigma^2} = \sqrt{16} = 4$

99.7% Range $\quad \mu \pm 3\sigma = 16 \pm 3(4) = 16 \pm 12$

$\Rightarrow \boxed{4 \text{ to } 28}$

$P(x = 10) = \text{poissonpdf}(16, 10) = \boxed{.034}$

$P(x < 20) = P(x \leq 19) = \text{Poissoncdf}(16, 19) = \boxed{.812}$

Apr 9-7:36 PM

Wendy meets 4 students in average per hour for consulting.

$\mu = 4$ fixed interval

$\mu = 4$ $\sigma^2 = \mu = 4$ $\sigma = \sqrt{\sigma^2} = 2$

68% Range $\rightarrow \mu \pm \sigma = 4 \pm 2 \rightarrow \boxed{2 \text{ to } 6}$

P(she meets at most 5 students)

$P(x \le 5) = \text{Poissoncdf}(4,5) = \boxed{.785}$

P(she meets at least 5 students)

$P(x \ge 5) = 1 - P(x \le 4)$

$= 1 - \text{Poissoncdf}(4,4)$

we don't want 4 | 5 we want

$= \boxed{.371}$

P(she meets between 2 to 6 students per hour, inclusive)

$P(2 \le x \le 6) = P(x \le 6) - P(x \le 1)$

$= \text{Poissoncdf}(4,6) - \text{Poissoncdf}(4,1)$

1 2 6

$= \boxed{.798}$

SG 17 ✓

Apr 9-7:43 PM

SG 14 – 17 Discrete Random Variable and Prob. dist.

SG 18 – 21 Continuous Random Variable and Prob. dist.

1) Uniform Prob. dist.

2) Standard normal Prob. dist.

3) Normal Prob. dist.

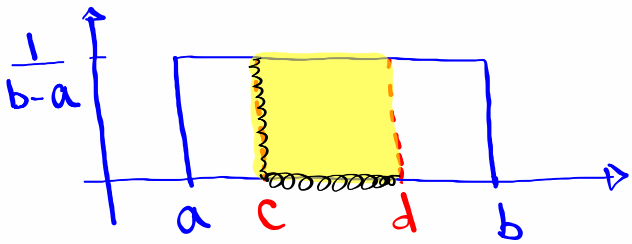
4) Central Limit Theorem

5) Applications

Apr 9-8:13 PM

Uniform Prob. dist.

Let x be a uniform Prob. dist. for all values from a to b.



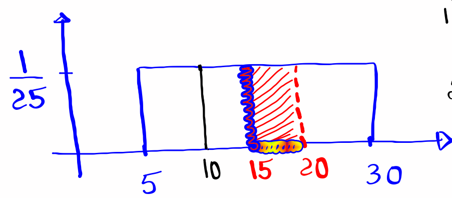
$P(x=c)=0$

$P(c<x<d)=$

$(d-c)\cdot\frac{1}{b-a}$

Apr 9-8:16 PM

Consider a uniform Prob. dist. for all values from 5 to 30.



1) $P(x=10)=0$

2) $P(15<x<20)$

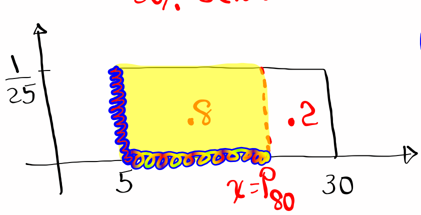
$=(20-15)\cdot\frac{1}{25}$

$=\frac{5}{25}=\boxed{\frac{1}{5}=}\boxed{.2}$

3) find $x=P_{80}$

80% below

20% above



$(x-5)\cdot\frac{1}{25}=.8$

Multiply by 25

$x-5=25(.8)$

$x-5=20\rightarrow\boxed{x=25}$

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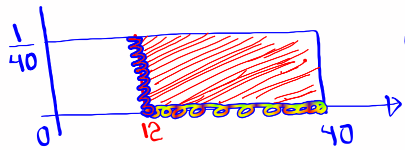
Consider a uniform Prob. dist. for all values from 0 to 40.

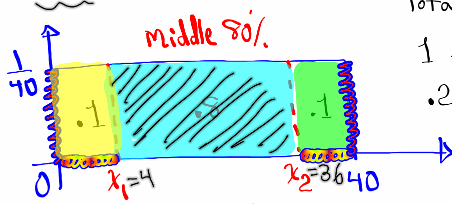


1) $P(x > 12) =$

$(40 - 12) \cdot \frac{1}{40}$

$= \frac{28}{40} = \frac{7}{10} = \boxed{.7}$

2) Find two values that separate the middle 80% from the rest.



Total Area = 1

$1 - .8 = .2$

$.2 \div 2 = .1$

$(x_1 - 0) \cdot \frac{1}{40} = .1$ $\quad x_1 = 40(.1) \rightarrow \boxed{x_1 = 4}$

$(40 - x_2) \cdot \frac{1}{40} = .1$ $\quad 40 - x_2 = 40(.1)$

$40 - x_2 = 4 \rightarrow \boxed{x_2 = 36}$

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Standard Normal Prob. dist.:

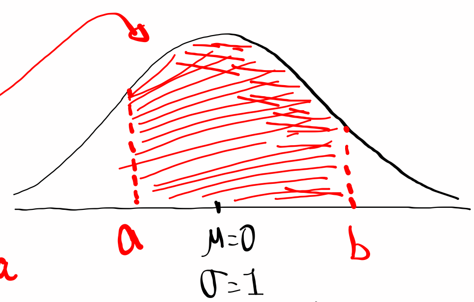
1) we use Z, $P(Z = c) = 0$

2) Data dist. is symmetric, bell-shape, with total area = 1

3) mean = mode = median

4) $\mu = 0$, $\sigma = 1$

5) $\boxed{P(a < Z < b)}$



How to find that area

$\boxed{2nd}$ $\boxed{VARS}$ normalcdf(L, U, μ, σ)

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$P(1 < Z < 1.75)$

$= \text{normalcdf}(1, 1.75, 0, 1)$

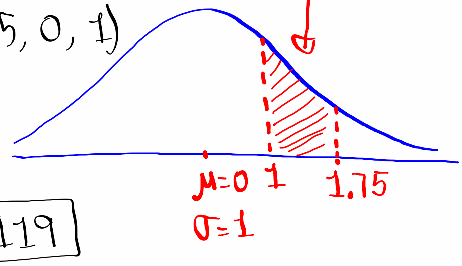
Lower: 1

Upper: 1.75

$\mu = 0$

$\sigma = 1$

$= \boxed{.119}$



$P(Z > -1.96)$

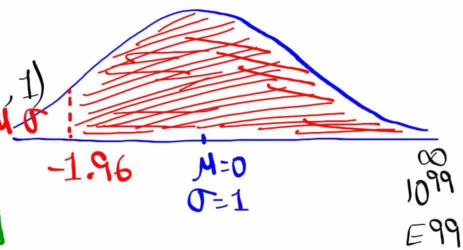
$= \text{normalcdf}(-1.96, E99, 0, 1)$

L U μ σ

(−)

$\boxed{2nd}\ \boxed{,}$

$= \boxed{.975}$



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$P(Z < 1.645)$

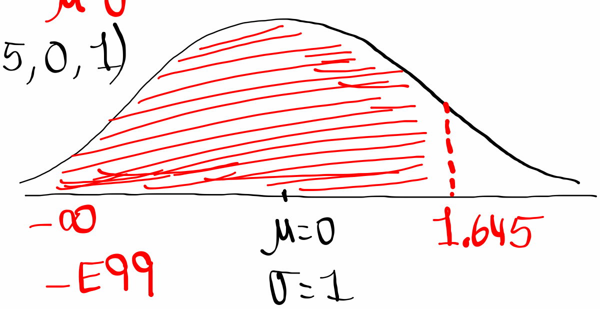
L U μ σ

$= \text{normalcdf}(-E99, 1.645, 0, 1)$

(−)

$\boxed{2nd}\ \boxed{,}$

$= \boxed{.950}$



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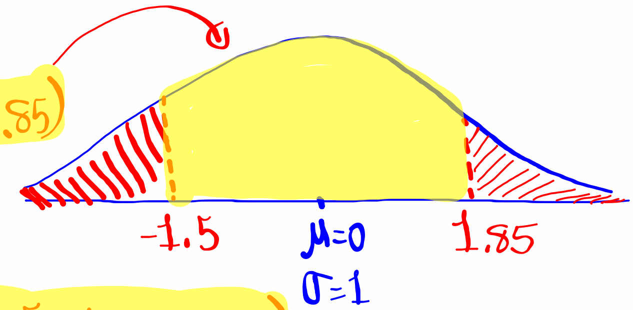
$P(Z < -1.5 \text{ OR } Z > 1.85)$

Total Area = 1

$= 1 - P(-1.5 < Z < 1.85)$



(−)

$= 1 - \text{normalcdf}(-1.5, 1.85, 0, 1)$

$= \boxed{.099}$

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Now doing reverse

Find $Z = P_{98}$

Left Area .98

Right Area .02

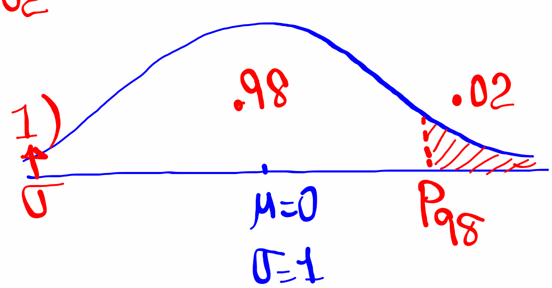
$Z = \text{invNorm}(.98, 0, 1)$

2nd VARS

Left Area

μ

σ



$= \boxed{2.054}$ ✓

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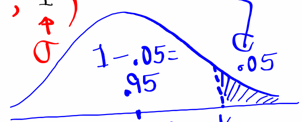
Find K Such that $P(Z > K) = .05$ Right Area

$K = \text{invNorm}(.95, 0, 1)$

.95 ↑ Left Area, 0 ↑ μ, 1 ↑ σ

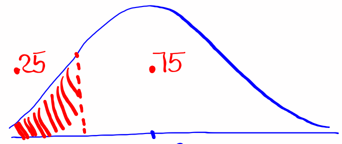
$= \boxed{1.645}$



Find $Z = Q_1$

25% below

75% above



$Q_1 = \text{invNorm}(.25, 0, 1) = \boxed{-.674}$

.25 Left Area, 0 μ, 1 σ

Drawing, Labeling, shading, and full TI Command required.

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Find two Z-values that Separate the middle 88% from the rest.

$1 - .88 = .12$

$.12 \div 2 = .06$

$Z_1 = P_6 = \text{invNorm}(.06, 0, 1)$

$= \boxed{-1.555}$

$Z_2 = P_{94} = \text{invNorm}(.94, 0, 1)$

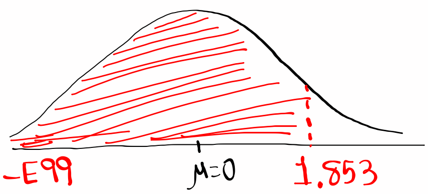
$= \boxed{1.555}$

Apr 9-9:17 PM

$N(0, 1)$

Normal Prob. dist. — μ — σ

Standard Normal Prob. dist.



$P(Z < 1.853)$

shade left

$\mu\ \sigma = 1$

$= \text{normalcdf}(\underset{L}{-E99}, \underset{U}{1.853}, \underset{\mu}{0}, \underset{\sigma}{1}) = \boxed{.968}$

(-)

$\boxed{2nd}\ \boxed{,}$